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Particle creation, renormalizability conditions and the mass–energy spectrum in gravity theories of quadratic Lagrangians

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Abstract. Massive scalar particle production, due to the anisotropic evolution of a five-dimensional spacetime, is considered in the context of a quadratic Lagrangian theory of gravity. Those particles, corresponding to field modes with non-vanishing momentum component along the fifth dimension, are created mostly in the neighbourhood of a singular epoch where only their high-frequency behaviour is of considerable importance. At the 1-loop approximation level, general renormalizability conditions on the physical quantities relevant to particle production are derived and discussed. Exact solutions of the resulting Klein–Gordon field equation are obtained and the mass–energy spectrum, attributed to the scalar field due to the cosmological evolution, is being investigated further. Finally, analytic expressions regarding the number and the energy density of the created particles at late times, are also derived and discussed.

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1. Introduction

Gravitational Lagrangian densities, which include higher-order curvature terms in connection to the Einstein–Hilbert (EH) one, are suggested by *superstring theories* (Candelas *et al* 1985, Green *et al* 1987) and by the 1-loop approximation of *quantum gravity* (Birrell and Davies 1982, Barrow and Ottewill 1983). Quadratic Lagrangians, in particular, have been used to yield renormalizable theories of gravity coupled to matter (Stelle 1977). They can also help us to improve the *semiclassical approximation*, where quantized matter fields interact with a classical gravitational field (Utiyama and De Witt 1962). In fact, renormalization of the energy–momentum tensor for a quantum field in four dimensions, indicates that the presence of quadratic terms in the gravitational action is expected *a priori* (Stelle 1977, 1978, Horowitz and Wald 1978).

However, every quadratic combination of curvature terms is not physically accepted, since its introduction into the gravitational action leads to differential equations of the fourth order with respect to the metric (Farina-Busto 1988) and those higher-derivative terms are associated with *ghost particles* (Weinberg 1979, Zwiebach 1985). A *ghosts-free*, nonlinear Lagrangian theory of gravity was formulated by Lovelock (1971). He proposed that the most general gravitational Lagrangian is of the form

$$\mathcal{L} = \sqrt{-g}\sum_{m=0}^{n/2}\lambda_m \mathcal{L}^{(m)} \qquad (1)$$

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