Cosmological solutions in Kaluza–Klein theories of quadratic Lagrangians

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We present a new class of solutions in a five-dimensional Gauss–Bonnet cosmology. The space-time consists of one time-like direction and two maximally symmetric space-like subspaces, the external space and the internal one. The universe is filled with matter in the form of a one-component perfect fluid and the analysis is carried out for various equations of state. The solutions to the field equations are based on a new approximation technique that determines the scales of the ordinary universe, in terms of the cosmological redshift parameter, at which the error between the exact and the approximate description becomes minimum, at each order of approximation. In this case, the four-dimensional Friedmann models of General Relativity result as external space solutions in the first-order approximation of the quadratic theory. Conditions for the existence of the resulting five-dimensional models are derived and discussed, together with their cosmological behavior. In this context, it is shown that under certain conditions, some of the models may be geodesically complete.
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I. INTRODUCTION

Recent work on theoretical cosmology is characterized by extending the four-dimensional cosmological models of General Relativity (GR), in two directions: (i) Kaluza–Klein (KK) models with more than four dimensions.[1] (ii) Models obtained from gravitational Lagrangians containing nonlinear terms in the curvature tensor.[2] If, in particular, the quadratic Gauss–Bonnett (GB) combination of the nonlinear terms is used, the resulting theory differs from GR only if the space-time has more than four dimensions. In this sense, a combination of these theories probably yields a natural generalization of GR in higher-dimensional space-times.

The idea that the space-time may have more than four dimensions was introduced by Kaluza[3] and Klein[4] in an effort to unify gravity and electromagnetism, an idea that was recently renewed by Schmützer,[5] without much success.[6] Higher-dimensional theories have been studied as an attractive way to unify all gauge interactions with gravity, in a *supergravity* scenario[7–9] and established as unavoidable in *superstring* theories.[10] There is now an extensive literature on different aspects of higher-dimensional cosmologies.[11–22] In any realistic theory the *extra* or *internal space* is assumed to be, at present, a compact manifold of very small size compared to that of the visible space.[11–14] This size is directly related to the fundamental constants and consequently must be stable,[13,14] something that leads to the problem of *compactification* of the extra dimensions.[14] In this context, it has been suggested that compactification of the extra space may be achieved in a natural way by adding a square curvature term, $\mathbb{R}_{\mu\nu\kappa\lambda}\mathbb{R}^{\mu\nu\kappa\lambda}$, in the action for the gravitational field.[23,24]

Gravitational Lagrangians containing nonlinear terms, were first considered in connection with Weyl's scale invariant theory of gravity.[25] Quadratic Lagrangians, in particular, have been studied classically in the search of solutions free from cosmological singularities,[26–33] but they attracted the interest of cosmologists only after it became clear that they could lead to inflation without phase transitions.[34,35]

It has been also proposed that quadratic gravitational Lagrangians are capable of yielding

