Higher-dimensional models in gravitational theories of quartic Lagrangians

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Ten-dimensional models, arising from a gravitational action which includes terms up to the fourth order in curvature tensor, are discussed. The space–time consists of one time direction and two maximally symmetric subspaces, filled with matter in the form of an anisotropic fluid. Numerical integration of the cosmological field equations indicates that exponential, as well as power law, solutions are possible. We carry out a dynamical study of the results in the $H_{\text{ext}}-H_{\text{int}}$ plane and confirm the existence of *attractors* in the evolution of the Universe. Those attracting points correspond to ''*extended*'' *De Sitter* space–times in which the external space exhibits inflationary expansion, while the internal one contracts. © *1997 American Institute of Physics.* [S0022-2488(97)01606-X]

I. INTRODUCTION

The mathematical background for a nonlinear Lagrangian theory of gravity was first formulated by Lovelock,[1] who proposed that the most general gravitational Lagrangian is

$$\mathscr{L}=\sqrt{-g}\sum_{m=0}^{n/2}\lambda_{(m)}\mathscr{L}_{(m)}, \tag{1.1}$$

where $\lambda_{(m)}$ are coupling constants. n denotes the manifold's dimensions. g is the determinant of the metric tensor, and $\mathscr{L}_{(m)}$ are functions of the Riemann curvature tensor of the form

$$\mathscr{L}_{(m)}=\frac{1}{2^m}\,\delta^{3_1\cdots\beta_{2m}}_{\alpha_1\cdots\alpha_{2m}}\mathscr{R}^{\alpha_1\alpha_2}_{\beta_1\beta_2}\cdots\mathscr{R}^{\alpha_{2m-1}\alpha_{2m}}_{\beta_{2m-1}\beta_{2m}}, \tag{1.2}$$

where δ^{α}_{β} is the Kronecker symbol. $\mathscr{L}_{(0)}$ is the volume n-form which gives rise to the cosmological constant, $\mathscr{L}_{(1)}=\frac{1}{2}\mathscr{R}$ is the Einstein–Hilbert (EH) Lagrangian, and $\mathscr{L}_{(2)}$ is the quadratic Gauss–Bonnett (GB) combination.[2] Euler variation of the gravitational action corresponding to Eq. (1.1) yields the most general symmetric and divergenceless tensor, which describes the propagation of the gravitational field and depends only on the metric and its first- and second-order derivatives.[1]

While quadratic Lagrangians have been widely studied (e.g., see Refs. 3 and 4 and references therein), cubic and/or quartic Lagrangians only recently have been introduced in the discussion of cosmological models in the framework of superstring theories.[5–10] The reason is that it is very hard to derive and (even harder) to solve the corresponding field equations. In this case solutions may be obtained only through certain numerical techniques,[11,12] where the idea of ''*attractor*'' plays a central role:[13] If some special space–time is the attractor for a wide range of initial conditions, such a space–time is naturally realized asymptotically. Since the ten-dimensional superstring theory is a candidate for a realistic unified theory, it is very important to investigate whether a similar attractor exists in this theory.

In the present paper we integrate numerically the field equations, resulting from a quartic gravitational Lagrangian, to obtain anisotropic, ten-dimensional cosmological models. The space–time consists of one time direction and two maximally symmetric subspaces, FRW⊗FRW: the

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