Conditions on the stability of the external space solutions in a higher-dimensional quadratic theory of gravity

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By using Lyapounov's direct method we examine the conditions under which stable solutions to the field equations for the scale function of the external space may be derived in the context of a five-dimensional quadratic theory of gravity. We show that the time evolution of the distance, in a diagram $t-R$, between our solution to the field equations and a neighbouring one is determined, in the linear approximation, in terms of a second-order linear differential equation. Asking for bounded solutions of this equation we arrive at a stability criterion for the external scale function solutions, indicating that there exist three types of cosmological evolution of the visible universe which are linearly stable at all times. These are (i) the Milne model, (ii) the spatially flat Friedmann radiation solution, and (iii) the De Sitter inflationary solution. © *1996 American Institute of Physics.* [S0022-2488(96)00708-6]

I. INTRODUCTION

The idea that the space–time may have more than four dimensions has been extensively studied, as an attractive way to unify all gauge interactions with gravity, in a *supergravity* scenario[1–3] and in *superstring* theories.[4] In a realistic higher-dimensional theory the extra dimensions are assumed to form, at present, a compact manifold (*internal space*) of very small size compared to that of the visible universe (*external space*),[5–8] leading to the problem of *compactification*.[8] It has been suggested that compactification of the extra space may be achieved in a natural way by adding a square curvature term, $\mathbb{R}_{\mu\nu\kappa\lambda}\mathbb{R}^{\mu\nu\kappa\lambda}$, in the action for the gravitational field.[9,10]

Gravitational Lagrangians containing quadratic terms of the curvature tensor have been studied classically in the search of inflationary solutions[11,12] and solutions free from cosmological singularities.[13–17] However, they attracted the interest of cosmologists only after it became clear that when gravity is extracted by the low energy approximation of superstrings, an additional $\mathbb{R}_{\mu\nu\kappa\lambda}\mathbb{R}^{\mu\nu\kappa\lambda}$ term arises in the gravitational action.[18,19]

The introduction of a quadratic combination into the gravitational action leads to differential equations of the fourth order with respect to the metric.[20,21] There is one particular combination of the quadratic curvature terms[22,23] which, in connection to the linear Einstein–Hilbert (EH) term, yields second-order differential equations. We refer to it as the Gauss–Bonnett (GB) combination, since in four dimensions it satisfies the relation

$$\frac{\delta}{\delta g_{\mu\nu}}\int(\mathbb{R}^2-4\mathbb{R}_{\mu\nu}\mathbb{R}^{\mu\nu}+\mathbb{R}_{\mu\nu\kappa\lambda}\mathbb{R}^{\mu\nu\kappa\lambda})\sqrt{-g}\,d^4x=0 \tag{1.1}$$

corresponding to the GB theorem.[24] Equation (1.1) implies that the addition of the GB combination to the EH Largangian will not affect the four-dimensional field equations. Therefore, the resulting theory differs from general relativity (GR) only if the space–time has more than four dimensions and, probably, yields a natural generalization of GR in higher-dimensional space–times.

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