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Geodesic motions versus hydrodynamic flows in a gravitating perfect fluid: dynamical equivalence and consequences

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Abstract. Stimulated by the methods applied for the observational determination of masses in the central regions of the active galactic nuclei (AGN), we examine, in the context of the theory of general relativity, the exact conditions, under which, in the interior of a gravitating perfect-fluid source, the geodesic motions and the adiabatic hydrodynamic flows are *dynamically equivalent* to each other. Dynamical equivalence rests on the functional similarity between the corresponding, covariantly expressed differential equations of motion and is obtained with the aid of a conformal transformation between the metric tensors of the original fluid, on the one hand, and the so-called *virtual fluid* on the other. In the latter, the hydrodynamic flow motions are formally the same as the geodesic motions. The conformal factor so obtained is written in terms of the specific enthalpy of the original fluid, and hence it is attributed a clear physical interpretation. The components of the virtual fluid's energy–momentum tensor are determined, through the invariant field equations, in terms of the original fluid's corresponding quantities, the conformal factor and its spacetime derivatives. In the Newtonian limit, the extra contribution to the original energy density results in an extra inertial-energy density and hence in an extra mass, both of which are always non-vanishing. The associated results indicate that, in the determination of the masses in the central regions of the AGNs, the *observationally determined nuclear mass* is being *underestimated* with respect to the real physical one. Accordingly, we evaluate the corresponding *mass deficit*, which, in typical cases of AGNs, is not always negligible compared with the mass of the central dark object, and it can be comparable to the total rest mass of the circumnuclear gas involved. Finally, the implications of the results are discussed, on the assumed form of the mass-density distribution law for the circumnuclear gas and the corresponding form of the extra inertial-energy density. We find that, under certain conditions, the density index is directly related to the polytropic index in the fluid's adiabatic equation of state.

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1. Introduction

It is generally believed that active galaxies are powered by the presence of central, massive black holes (Rees 1984, Blandford and Rees 1992), and therefore, massive black holes are expected to be common in any *active-galactic-nucleus* (AGN) phenomenon (Holt *et al* 1992, Antonucci 1993, Urry and Padovani 1995). Moreover, black holes are expected to be present in the centres of many quiescent galaxies as well (Chokshi and Turner 1992). However, direct dynamical evidence for the presence of black holes in the central regions of individual galaxies is scarce. Stellar kinematical studies have provided tentative evidence for black holes only in a handful of nearby galaxies, because of difficulties in spatial resolution and the lack of

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