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Interaction of charged particles with gravitational waves of various polarizations and directions of propagation

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Abstract. The interaction of a charged particle, moving in a uniform magnetic field, with a gravitational wave is considered for two types of wave polarization (linear and circular) and for an arbitrary direction of propagation, with respect to the magnetic field. It is found that, in the oblique and perpendicular propagation cases, the motion of the particle is chaotic. An asymptotic criterion is calculated, which determines, for both types of polarization, the stochasticity threshold in terms of the wave amplitude and frequency as well as the initial momentum of the particle. We also show that, under a resonant condition, we can find the exact solutions of the geodesics in the case of parallel propagation. The solutions are unbounded, leading to an "infinite" acceleration of the charged particle. The particle energisation through either the chaotic or the resonant interaction may lead to a damping of the gravitational wave in the interstellar space.

Key words: gravitation – acceleration of particles

1. Introduction

In a previous paper (Varvoglis & Papadopoulos 1992, which hereafter is referred to as Paper I), the interaction between a charged particle in a uniform magnetic field, $\boldsymbol{B}=B_0\boldsymbol{e}_z$, and a plane linear polarized gravitational wave, propagating perpendicular to the direction of the magnetic field, has been studied. It was found that for certain values of the amplitude of the wave, α, and the relative frequency, $\nu=\omega/\Omega$ (where by ω we denote the wave's angular frequency and by Ω the Larmor angular frequency of the particle), the motion of the particle becomes chaotic. In this way, secular transfer of energy between the wave and the particle is possible, resulting in a possible damping of the wave. In the present paper we consider the same model, but for two cases of polarization (linear and circular) and for an arbitrary direction of propagation of the gravitational wave (i.e. perpendicular, oblique or parallel).

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The motion of a charged particle in a curved space-time is given, in Hamiltonian formalism (Misner et al. 1973), by the differential equations

$$\frac{\mathrm{d}x^\mu}{\mathrm{d}\lambda}=\frac{\partial H}{\partial \pi_\mu}, \qquad \frac{\mathrm{d}\pi_\mu}{\mathrm{d}\lambda}=-\frac{\partial H}{\partial x^\mu}, \tag{1}$$

where π_μ are the generalized momenta (corresponding to the coordinates x^μ) and the "super Hamiltonian", H is given, in a system of geometrical units ($h=c=G=1$), by the relation

$$H=\tfrac{1}{2}g^{\mu\nu}(\pi_\mu-eA_\mu)(\pi_\nu-eA_\nu)\equiv\tfrac{1}{2}. \tag{2}$$

In Eq. (2) $g^{\mu\nu}$ is the contravariant metric tensor, which is defined as

$$g^{\mu\nu}=\eta^{\mu\nu}+h^{\mu\nu},$$

with $\eta^{\mu\nu}=\mathrm{diag}\,(1,\,-1,\,-1,\,-1)$ and $|h^{\mu\nu}|\ll 1$. A_μ is the vector potential, corresponding to the tensor of the electromagnetic field in a curved space-time, $F_{\mu\nu}$. The mass of the particle is taken equal to 1. For the specific form of the magnetic field, $\boldsymbol{B}=B_0\boldsymbol{e}_z$, we have

$$A^0=A^1=A^3=0, \qquad A^2=B_0x^1.$$

2. Oblique propagation

2.1 Linear polarized gravitational wave

In the case of a linear polarized wave, the only non-zero components of the metric tensor are (Ohanian 1976; Papadopoulos & Esposito 1981)

$$g^{00}=1, \qquad g^{11}=\frac{1-\alpha\sin^2\theta\cos k_\mu x^\mu}{-1+\alpha\cos k_\mu x^\mu},$$

$$g^{22}=-\frac{1}{1+\alpha\cos k_\mu x^\mu},$$

$$g^{33}=\frac{1-\alpha\cos^2\theta\cos k_\mu x^\mu}{-1+\alpha\cos k_\mu x^\mu},$$

$$g^{13}=g^{31}=-\frac{\alpha}{2}\frac{\sin 2\theta\cos k_\mu x^\mu}{-1+\alpha\cos k_\mu x^\mu},$$